Differential Equations

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1 Daily Quiz

2 Key Topics

Today, we use the Laplace transform to solve several initial value problems of the form

$$ay'' + by' + cy = f(t),\ y(0) = y_0,\ y(0) = y_0', \tag{1}$$

where $f(t)$ is piecewise continuous and exponentially bounded. For further reading, see [1, Sections 8.3 and 8.5].

Let $Y(s)$ denote the Laplace transform of $y(t)$ and $F(s)$ denote the Laplace transform of $f(t)$. Then, we can transform (1) to the following form

$$Y(s)\left(as^2 + bs + c\right) = F(s) + (as + b)y(0) + ay'(0).$$

Therefore, we can solve for $Y(s)$. If we can find $y(t) = \mathcal{L}^{-1}(Y(s))$, then we have found the unique solution to (1).

3 Exercises

Use the Laplace transform to solve the following initial value problems.

1. $y'' + 4y = 3\sin(t),\ y(0) = 1,\ y'(0) = -1.$
2. $y'' - 2y' = \begin{cases} 4 & 0 \leq t < 1 \\ 6 & t \geq 1 \end{cases},\ y(0) = -6,\ y'(0) = 1$
3. $y'' + 2y' + 2y = \begin{cases} \cos(t) & 0 \leq t < \pi \\ 0 & t \geq \pi \end{cases},\ y(0) = 1,\ y'(0) = 1$

References

[1] W. Trench, *Elementary Differential Equations with Boundary Value Problems*, Creative Commons Attribution-Noncommercial-Share Alike, 1st ed., 2013.