Differential Equations

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1 Daily Quiz

Find a fundamental set of solutions to the homogeneous differential equation:

$$y''' + 3y'' + 4y' + 2y = 0,$$

given that the characteristic equation can be factored as follows:

$$r^3 + 3r^2 + 4r + 2 = (r+1)(r^2 + 2r + 2).$$

2 Key Topics

Today, we discuss the method of undetermined coefficients for finding a particular solution a higher-order non-homogeneous differential equation of the form:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = f(t). \tag{1}$$

where $a_n \neq 0$. For further reading, see [1, Section 9.3].

2.1 Undetermined Coefficients

As with second-order differential equations, the method of undetermined coefficients can only be applied to an $f(t)$ of a form in the following table.

$f(t)$	$y_p(t)$
$\sum_{i=0}^{n} a_i t^i$	$\sum_{i=0}^{n} A_i t^i$
$\left(\sum_{i=0}^{n} a_i t^i\right) e^{at}$	$\left(\sum_{i=0}^{n} A_i t^i\right) e^{at}$
$\left(\sum_{i=0}^{n} a_i t^i\right) e^{at}\cos(bt)$	$\left(\sum_{i=0}^{n} A_i t^i\right) e^{at}\cos(bt) + \left(\sum_{i=0}^{n} B_i t^i\right) e^{at}\sin(bt)$
$\left(\sum_{i=0}^{n} a_i t^i\right) e^{at}\sin(bt)$	$\left(\sum_{i=0}^{n} A_i t^i\right) e^{at}\cos(bt) + \left(\sum_{i=0}^{n} B_i t^i\right) e^{at}\sin(bt)$

3 Exercises

Find the general solution for each of the following non-homogeneous differential equations.

I. $y''' + 3y'' + 4y' + 2y = 3t^2 + 4t + 1$

II. $y''' + 3y'' + 4y' + 2y = 3e^t + 4\cos(t)$

III. $y''' + 3y'' + 4y' + 2y = e^{-t}$

References

[1] W. Trench, *Elementary Differential Equations with Boundary Value Problems*, Creative Commons Attribution-Noncommercial-Share Alike, 1st ed., 2013.