Differential Equations

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1 Daily Quiz

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} -1 & 2 \\ -6 & 6 \end{bmatrix}.$$

2 Key Topics

Today we review for Exam 1, which will cover solving first-order differential equations using integration factor, separation of variables, and the theory of exact differential equations. In addition, Exam 1 will cover existence and uniqueness of solutions and the bifurcation diagram.

3 Exercises

I. Find the general solution of

$$ty' = \sqrt{1-y^2}.$$

II. Find the general solution of

$$y' - 5y = t.$$

III. Sketch the bifurcation diagram for

$$y' = \lambda - 3y + y^2.$$

IV. Which of the following initial value problems are guaranteed to have a unique solution? Justify your answer.

a. $y' = \sqrt{y},\ y(1) = 0$
b. $y' = \sqrt{y},\ y(1) = 1$
c. $y' = \frac{t}{y-2},\ y(2) = 0$
d. $y' = \frac{y}{t} + 2t,\ y(0) = 1$

V. Find the general solution of

$$\left(4t^3y^3 + 3t^2\right) + \left(3t^4y^2 + 6y^2\right)y' = 0$$

VI. Find the general solution of

$$y + \left(2t + \frac{1}{y}\right)y' = 0.$$