

Real Analysis

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1 Daily Quiz

Let $f: S \rightarrow \mathbb{R}$. State the definition of f being uniformly continuous on S .

2 Key Topics

Today we finish our discussion uniformly continuous functions. For further reading, see [1, Section 3.4]. Note that [1] refers to an accumulation point as a cluster point.

Last time, we saw that $f(x) = x^2$ is not uniformly continuous on \mathbb{R} . Today, we show that uniform continuity implies continuity. Then, we show that continuity over a compact set is sufficient for uniform continuity on that set.

2.1 Uniform Continuity

Proposition 2.1. *Let $f: S \rightarrow \mathbb{R}$. If f is uniformly continuous on S , then f is continuous at all $c \in S$.*

Theorem 2.2. *Suppose $f: S \rightarrow \mathbb{R}$ is continuous. If S is compact, then f is uniformly continuous.*

3 Exercises

- I. Prove Theorem 2.2.
- II. Prove Lemma 2.4.
- III. Prove Theorem 2.5.
- IV. Prove Corollary 2.6.

References

- [1] J. LEBL, *Basic Analysis: Introduction to Real Analysis*, Creative Commons Attribution-Noncommercial-Share Alike, 6th ed., 2023.