Real Analysis

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1 Daily Quiz

Let $S = (1, 2) \cup (2, 3]$. Find the interior, boundary, and accumulation points of S.

2 Key Topics

On 9/13/2023 – 9/15/2023, we introduced the notion of bounded subsets and on 9/18/2023 introduced open and closed sets. Today, we continue our investigation of the real numbers by introducing compact sets, which are open subsets of $\mathbb{R}$ that are both closed and bounded. For further reading, see [1, Section 1.3].

2.1 Compact Sets

Definition 2.1. Let $S \subseteq \mathbb{R}$ and let $\mathcal{F}$ be a set of open sets. We say that $\mathcal{F}$ is an *open cover* of S if

$$S \subseteq \bigcup_{A \in \mathcal{F}} A.$$

If $\mathcal{G} \subseteq \mathcal{F}$ is an open cover of S, then we say that $\mathcal{G}$ is a *subcover* of S.

Definition 2.2. Let $S \subseteq \mathbb{R}$. We say that S is compact if and only if every open cover of S contains a finite subcover.

Lemma 2.3. *If S is a nonempty closed bounded subset of $\mathbb{R}$, then S has a maximum and a minimum.*

Theorem 2.4 (Heine-Borel)**.** *A subset S of $\mathbb{R}$ is compact if and only if S is closed and bounded.*

3 Exercises

I. Prove Lemma 2.3

II. Prove Theorem 2.4

References

[1] W. Trench, *Introduction to Real Analysis*, Creative Commons Attribution-Noncommercial-Share Alike, 2nd ed., 2013.