Real Analysis

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1 Daily Quiz

State the definition of convergence for $s\colon \mathbb{N} \to \mathbb{R}$.

2 Key Topics

Having established the limit of a sequence:

$$\lim_{n\to\infty} s_n = L,$$

if $\forall \epsilon > 0,\ \exists N \in \mathbb{R} \ \ni \ n > N \ \Rightarrow \ |s_n - L| < \epsilon$, we now establish basic arithmetic properties of the limit. Today, we break into groups and prove the following theorem.

Theorem 2.1. *Let $s\colon \mathbb{N} \to \mathbb{R}$ and $t\colon \mathbb{N} \to \mathbb{R}$ be convergent with limits L and L', respectively. Then, the following hold*

a. $\lim_{n\to\infty} (s_n + t_n) = L + L'$,

b. $\lim_{n\to\infty} (ks_n) = kL$, *for any* $k \in \mathbb{R}$,

c. $\lim_{n\to\infty} (s_n \cdot t_n) = L \cdot L'$,

d. $\lim_{n\to\infty} \left(\frac{s_n}{t_n}\right) = \frac{L}{L'}$, *if* $L' \neq 0$.

For further reading, see [1, Section 2.2].

3 Exercises

I. Prove Theorem 2.1.

References

[1] J. Lebl, *Basic Analysis: Introduction to Real Analysis*, Creative Commons Attribution-Noncommercial-Share Alike, 6th ed., 2023.