Proof Techniques Worksheet

Real Analysis

Name: ______________________________ **Date:** ____________

1. Negating quantified statements.

Write the logical negation of each statement. Your answer should move the negation through every quantifier and connective; do not begin your final answer with "It is not true that."

(a) There exists a rational number whose square is equal to 2:

$$\exists x \in \mathbb{Q} \text{ such that } x^2 = 2.$$

(b) For every real number, there exists a larger real number:

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R} \text{ such that } y > x.$$

(c) There exists $\delta > 0$ such that every $x \in \mathbb{R}$ satisfies

$$|x - 1| < \delta \quad \Longrightarrow \quad |(2x + 3) - 5| < 1.$$

2. Contrapositive and parity.

Consider the statement:

If n^2 is even, then n is even, where $n \in \mathbb{Z}$.

(a) Write its converse, inverse, and contrapositive. Indicate which of these statements is logically equivalent to the original implication.

(b) Prove the original statement by proving its contrapositive. Your proof should use the definitions of even and odd integers.

3. Constructing a direct proof.

Prove the following statement directly:

> For every $\varepsilon > 0$, there exists $\delta > 0$ such that, for every $x \in \mathbb{R}$,
> $$|x - 2| < \delta \quad \Longrightarrow \quad |(3x + 1) - 7| < \varepsilon.$$

Before writing the proof, work backward from the desired inequality to determine an appropriate choice of δ. In the finished proof, proceed forward: let $\varepsilon > 0$ be arbitrary, state your choice of δ, and verify that it works. Make the order of the quantifiers clear.