

Sequences Definitions Quiz

Math 312: Concepts of Real Analysis

Name: _____

Date: _____

Directions. You have 20 minutes. State definitions precisely, including all quantifiers. Class notes and electronic devices are not permitted.

1. **Topological definitions.**

Let $S \subseteq \mathbb{R}$.

- (a) Define an interior point and a boundary point of S .
- (b) Define an accumulation point and an isolated point of S .
- (c) Define an open set and a closed set in $\mathbb{R}$.

2. Compactness and covers.

Let $K \subseteq \mathbb{R}$.

- (a) Define an open cover of K and a finite subcover of that cover. Make clear which sets may be selected.
- (b) Define compactness using open covers and finite subcovers.
- (c) State the Heine–Borel characterization of compact subsets of $\mathbb{R}$.

3. Convergence, Cauchy sequences, and monotonicity.

Let (s_n) be a real sequence and $L \in \mathbb{R}$.

- (a) State the definition of $\lim_{n \rightarrow \infty} s_n = L$.
- (b) Negate the definition in part (a): state precisely what it means that (s_n) does not converge to this specified L .
- (c) State the definition of a Cauchy sequence.
- (d) State the definitions of an increasing (nondecreasing) sequence and a decreasing (nonincreasing) sequence.
- (e) State the Cauchy convergence criterion for real sequences.