

Topology and Compactness Quiz

Math 312: Concepts of Real Analysis

Name: _____

Date: _____

Directions. You have 20 minutes. Show enough work to support each answer. You may use your class notes. No electronic devices are permitted.

1. Topological definitions and closure.

- (a) State the definitions of an interior point and an accumulation point of a set $S \subseteq \mathbb{R}$, including all quantifiers.
- (b) State the definitions of an open set and a closed set in $\mathbb{R}$.
- (c) Let

$$S = \left\{ \frac{2}{n} : n \in \mathbb{N} \right\} \cup \{3\}, \quad \mathbb{N} = \{1, 2, 3, \dots\}.$$

Is S closed? Is $\text{cl}(S)$ compact? Justify both answers.

2. Compactness and open covers.

- (a) State the definition of a compact set using open covers and finite subcovers. Make clear which sets may be selected for a subcover.
- (b) For $n \in \mathbb{N}$, let $U_n = (1/(n+1), 2)$. Show that $\mathcal{U} = \{U_n : n \in \mathbb{N}\}$ is an open cover of $(0, 1]$.
- (c) Does this cover have a finite subcover? Is $(0, 1]$ compact? Justify your answers.

3. Heine–Borel and extreme points.

- (a) State the Heine–Borel theorem for subsets of $\mathbb{R}$.
- (b) Let $K \subseteq \mathbb{R}$ be compact and nonempty. Prove that $\sup K$ exists and belongs to K .