Math-456 · HW 3

I. The result follows from the fact that orthogonal matrices are length preserving:

$$\kappa_2(QA) = \frac{\max_{\|x\|=1} \|QAx\|}{\min_{\|x\|=1} \|QAx\|} = \frac{\max_{\|x\|=1} \|Ax\|}{\min_{\|x\|=1} \|Ax\|} = \kappa_2(A)$$

II. Suppose that $Qx = y$, where $x, y \in \mathbb{R}^n$ are distinct vectors of the same length and $Q = I - \gamma u u^T$ for some non-zero vector u.

Then,

$$\begin{aligned} Qx = y &\Rightarrow (I - \gamma u u^T)x = y \\ &\Rightarrow x - \gamma u u^T x = y \\ &\Rightarrow x - y = \gamma u u^T x \\ &\Rightarrow \frac{1}{\gamma u^T x}(x - y) = u \end{aligned}$$

Hence, u is a constant multiple of $x - y$.

Let Q_c denote the reflector created by cu, i.e., a constant multiple of u. Then,

$$\begin{aligned} Q_c &= I - \frac{2}{\|cu\|_2^2}(cu)(cu)^T \\ &= I - \frac{2}{c^2\|u\|_2^2} c^2 u u^T \\ &= I - \frac{2}{\|u\|_2^2} u u^T = Q \end{aligned}$$

Therefore, the reflector created by any constant multiple of $(x-y)$ results in the same reflector.

Math-456-HW5

III. QA is computed by taking Q times each column of A. Note that

$$Qx = (I - \gamma u u^T)x$$
$$= x - \gamma u u^T x$$

$u^T x$ costs n multiplications and $(n-1)$ additions

$\gamma(u^T x)$ costs 1 multiplication

$\gamma(u^T x)u$ costs n multiplications

$x - \gamma(u^T x)u$ costs n subtractions.

Hence, Qx costs $4n$ flops.

Since A has m columns it follows that QA costs $4nm$ flops.

IV.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}[x] = \begin{bmatrix} 9 \\ 5 \\ 3 \\ 1 \end{bmatrix}$$

a. "Guess" is the average of the values in [

i.e., $\frac{9+5+3+1}{4} = \frac{18}{4} = \frac{9}{2}$.

Math-456-HW3

Also, since $\begin{bmatrix} 9 \\ 5 \\ 3 \\ 1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

it follows that $Q \begin{bmatrix} 9 \\ 5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 3 \\ 1 \end{bmatrix}$.

c. Therefore, the given linear system can be transformed to

$$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} [x] = \begin{bmatrix} 9 \\ 5 \\ 3 \\ 1 \end{bmatrix}$$

which has solution $x = 9/2$.