Math 482
Spring 2026
Exam 1
February 9, 2026

Name: ____________________

Pledge: ____________________

Each question topic and point value is recorded in the tables below. Note that this exam must be completed within the 50 minutes allotted during class. Also, you must work without any external resources (no notes nor calculator). You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	8	
2	8	
3	12	
4	12	
5	12	
Total:	52	

Topics Table

Question	Topic
1	Standard Form
2	Duality and Weak Duality
3	Simplex Tableaux and Pivoting
4	Auxiliary Method and Infeasibility
5	Geometry: Extreme Points and BFS

1. Consider the following optimization problem.

$$\begin{aligned} \text{minimize} \quad & z = 3x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 2,, \\ & -x_1 + 2x_2 \leq 4,, \\ & x_1 \leq 5,, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(a) (4 points) Rewrite this problem in standard form.

(b) (4 points) Draw the feasible region of the original problem in the (x_1, x_2)-plane. Clearly label each boundary line and shade the feasible region.

2. Consider the primal LP

$$\begin{aligned} \text{maximize} \quad & z = 2x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 6,, \\ & 2x_1 + x_2 \leq 8,, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(a) (4 points) Write the dual LP.

(b) (4 points) Prove weak duality for this particular primal–dual pair.

3. Consider the LP

$$\begin{aligned} \text{maximize} \quad & z = 5x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 6,, \\ & x_1 + 3x_2 \le 9,, \\ & x_1, x_2 \ge 0. \end{aligned}$$

(a) (4 points) Introduce slack variables and write the initial tableau.

(b) (4 points) Using the *least subscript rule*, perform one simplex pivot starting from your initial tableau.

(c) (4 points) For your new tableau, state the basis β, the non-basic variables π, the basic solution, and the objective value.

4. Consider the LP

$$\begin{aligned} \text{maximize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \leq -2,, \\ & -x_2 \leq -3,, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(a) (4 points) Introduce slack variables, write the initial tableau, and show that it is infeasible.

(b) (4 points) Write the corresponding auxiliary LP (Phase I problem).

(c) (4 points) Explain why the auxiliary method is always feasible and how the auxiliary method determines if the primal LP is feasible.

5. Let $P \subset \mathbb{R}^n$ be the feasible region of a linear program in standard form.

   (a) (4 points) Define what it means for a point $x \in P$ to be an extreme point of P.

   (b) (4 points) Explain why every feasible bounded linear program has an optimal solution attained at an extreme point of its feasible region.

   (c) (4 points) Explain why there is a one-to-one correspondence between extreme points of P and feasible basic solutions of the standard-form LP.