Math 482 **Name:** ______________________

Spring 2026

Exam 2

March 6, 2026 **Pledge:** ______________________

Each question topic and point value are recorded in the tables below. Note that this exam must be completed within the 50 minutes allotted during class. Also, you must work without any external resources (no notes nor calculator). You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	12	
2	10	
3	10	
4	10	
5	8	
Total:	50	

Topics Table

Question	Topic
1	Strong Duality and Reduced Cost
2	Complementary Slackness
3	Matrix Form: Simplex Method
4	Matrix Form: Optimality
5	Models on Graphs

1. Consider the primal LP

$$
\begin{aligned}
\text{maximize} \quad & z = 4x_1 + x_2 + 3x_3 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 \le 6, \\
& 2x_1 + x_2 + x_3 \le 8, \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

(a) (3 points) Write the dual LP.

(b) (6 points) An optimal tableau for the primal LP is shown below.

$$
\begin{array}{ccc|ccc|c}
1 & -1 & 0 & -1 & 1 & 0 & 2 \\
0 & 3 & 1 & 2 & -1 & 0 & 4 \\
\hline
0 & 4 & 0 & 2 & 1 & 1 & 20
\end{array}
$$

Use the reduced costs to identify the dual vector $\mathbf{y}^*$.

(c) (3 points) Verify strong duality.

2. Consider the primal LP

$$\begin{aligned} \text{maximize} \quad & z = 5x_1 + 3x_2 + 0x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \le 4, \\ & 2x_1 + x_2 \le 5, \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Suppose an optimal dual vector is $\mathbf{y}^* = (1, 2)^T$.

(a) (2 points) Compute the dual slack vector for each dual constraint at $\mathbf{y}^*$.

(b) (6 points) Using complementary slackness, determine an optimal primal vector $\mathbf{x}^*$.

(c) (2 points) Verify optimality using the complementary slackness conditions.

3. Consider the LP

$$\begin{aligned} \text{maximize} \quad & z = 2x_1 + 3x_2 + x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \leq 5, \\ & 2x_1 + x_2 \leq 6, \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(a) (4 points) Introduce slack variables and write the constraints in matrix form

$$A\mathbf{x}_\pi + I\mathbf{x}_\beta = \mathbf{b}, \qquad \mathbf{x} \geq 0.$$

Clearly identify A, $\mathbf{b}$, $\mathbf{x}_\pi$, and $\mathbf{x}_\beta$.

(b) (6 points) Consider the basis

$$\beta = \{2, 4\}, \qquad \pi = \{1, 3, 5\}.$$

(i) Write the matrices B and Π (as columns of $[A|I]$).

(ii) Compute the basic solution $\mathbf{x}_\beta = B^{-1}\mathbf{b}$.

4. Consider the LP from Question 3. Suppose the basis $\beta = \{1, 2\}$ yields the basic solution

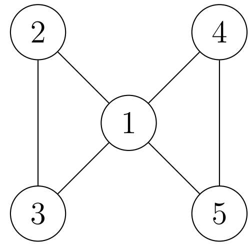
$$\mathbf{x} = (1, 4, 0, 0, 0).$$

(a) (4 points) Compute the dual vector $\mathbf{y}$ satisfying $B^T \mathbf{y} = \mathbf{c}_\beta$.

(b) (4 points) Compute the primal and dual slack vectors. Is the dual vector feasible?

(c) (2 points) Check the complementary slackness conditions. Are the solutions optimal?

5. Consider the graph below.



(a) (4 points) Write an integer programming model for the minimum vertex cover problem on this graph. Clearly define the decision variables.

(b) (4 points) Consider the subsets

$$F_1 = \{2, 3\}, \qquad F_2 = \{4, 5\}, \qquad F_3 = \{1, 2, 4\}, \qquad F_4 = \{1, 3, 5\}.$$

Write an integer programming model that selects a maximum number of these subsets such that no vertex appears in more than one chosen subset.