Homework 02 — Solutions

Math 482: Mathematical Methods of Operations Research (Spring 2026)
Weeks 2–3 (Jan 21–Jan 30, 2026)

Relevant topics: Simplex Method, Auxiliary Method, Infeasibility and Unboundedness.

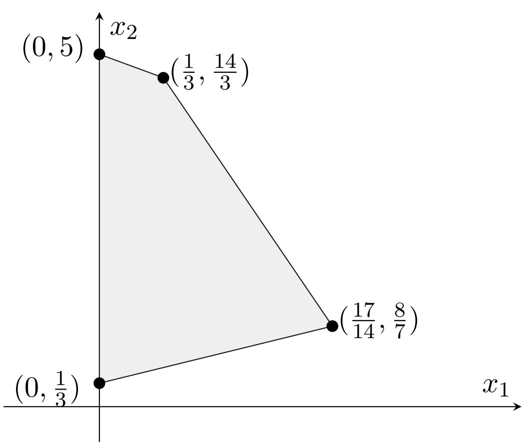
Due: Friday February 6, 2026.

Instructions: Show your work clearly; when working through the simplex method, clearly label the basis, non-basic variables, and basic solution.

I. Consider the primal LP shown below.

$$\begin{aligned} \text{maximize} \quad & z = 4x_1 + 5x_2 \\ \text{subject to} \quad & 2x_1 - 3x_2 \le -1, \\ & 4x_1 + x_2 \le 6, \\ & x_1 + x_2 \le 5, \\ & x_i \ge 0, \ \forall i \in \{1, 2\} \end{aligned}$$

a. **Solution.**



b. **Solution.** Introduce slack variables to write the system as equalities:

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= -1, \\ 4x_1 + x_2 + x_4 &= 6, \\ x_1 + x_2 + x_5 &= 5, \qquad x_j \ge 0. \end{aligned}$$

The initial tableau is

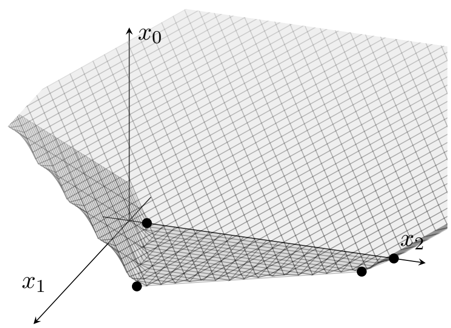
2	−3	1	0	0	−1
4	1	0	1	0	6
1	1	0	0	1	5
−4	−5	0	0	0	0

The corresponding basic solution is $x_1 = x_2 = 0$, $x_3 = -1$, $x_4 = 6$, $x_5 = 5$, which is infeasible. Hence, the initial tableau is infeasible.

c. **Solution.**

$$\begin{aligned} \text{maximize} \quad & v = -x_0 \\ \text{subject to} \quad & -x_0 + 2x_1 - 3x_2 \le -1, \\ & -x_0 + 4x_1 + x_2 \le 6, \\ & -x_0 + x_1 + x_2 \le 5, \\ & x_i \ge 0, \ \forall i \in \{0,1,2\} \end{aligned}$$

d. **Solution.**



e. **Solution.** Using the auxiliary method (Phase I), we obtain a feasible tableau for the primal LP. One Phase I endpoint is

$$(x_1, x_2) = (0, \tfrac{1}{3}).$$

f. **Solution.** Plot $(0, \frac{1}{3})$ on the sketch from part (d).

g. **Solution.** Starting from a feasible tableau, Phase II simplex gives the path

$$(0, \tfrac{1}{3}) \to \left(\tfrac{17}{14}, \tfrac{8}{7}\right) \to \left(\tfrac{1}{3}, \tfrac{14}{3}\right) \to (0, 5),$$

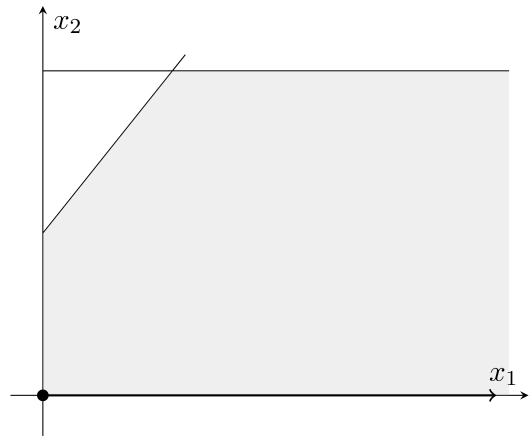
and the optimal solution is $(x_1, x_2) = (0, 5)$ with $z = 25$.

h. **Solution.** Plot the Phase II points on the sketch from part (a) and connect successive points along edges.

II. Consider the following primal LP.

$$\begin{aligned} \text{maximize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \le 2, \\ & x_2 \le 4, \\ & x_i \ge 0, \ \forall i \in \{1,2\} \end{aligned}$$

(a) **Solution.**
The constraint $-x_1 + x_2 \le 2$ is equivalent to $x_1 \ge x_2 - 2$. Together with $0 \le x_2 \le 4$, the feasible region is an infinite strip to the right. The ray $\{(t, 0) : t \ge 0\}$ is feasible, so the region is unbounded.



(b) **Solution.** Add slacks x_3, x_4:

$$-x_1 + x_2 + x_3 = 2, \qquad x_2 + x_4 = 4, \qquad x \geq 0.$$

The initial tableau is

-1	1	1	0	2
0	1	0	1	4
-1	-1	0	0	0

The reduced cost of x_1 is negative, so x_1 enters. Since the x_1 column has no positive entries in the constraint rows, the minimum ratio test fails. Hence, the LP is unbounded.

(c) **Solution.** Take a feasible point $x = (0, 0)$ and direction $d = (1, 0)$. Then for every $t \geq 0$, $x + td = (t, 0)$ is feasible and $z(x + td) = t \to \infty$, so d is an unbounded direction.