Homework 03

Math 482: Mathematical Methods of Operations Research (Spring 2026)
Week 6 (Feb 16–Feb 20, 2026)

Relevant topics: Primal–Dual Relationship, Strong Duality, Complementary Slackness.
Due: Friday February 20, 2026.
Instructions: Show your work clearly. When using the simplex method, clearly indicate your pivot choices (least subscript rule) and the corresponding basic feasible solutions.

I. Consider the following primal linear program.

$$\begin{aligned} \text{maximize} \quad & z = 3x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 4, \\ & 2x_1 + x_2 \leq 5, \\ & x_1 + 3x_2 \leq 7, \\ & x_i \geq 0, \ \forall i \in \{1, 2\} \end{aligned}$$

(a) Write the dual linear program.
(b) Solve the primal problem.
(c) Use complementary slackness to find an optimal dual solution and verify strong duality.

II. Consider the primal linear program

$$\begin{aligned} \text{maximize} \quad & z = 6x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 8, \\ & 2x_1 + x_2 \leq 10, \\ & x_i \geq 0, \ \forall i \in \{1, 2\} \end{aligned}$$

(a) Introduce slack variables and write the initial tableau.
(b) Using the simplex method (least subscript rule), compute an optimal tableau.
(c) From your optimal tableau, read off a dual optimal solution and verify complementary slackness.

III. Consider the primal linear program

$$\begin{aligned} \text{maximize} \quad & z = 6x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 8, \\ & 2x_1 + x_2 \leq 10, \\ & x_i \geq 0, \ \forall i \in \{1, 2\} \end{aligned}$$

(a) Draw the feasible region of the primal problem, indicate the optimal solution.
(b) Draw the feasible region of the dual problem, indicate the optimal solution.
(c) The dual optimal solution provides an optimal conic combinaion of the primal constraints. Write this combination (where the inequalities are replaced by equalities). Draw this equation on your graph from part (a).
(d) The primal optimal solution provides an optimal conic combination of the dual constraints. Write this combination (where the inequalities are replaced by equalities). Draw this equation on your graph from part (b).