

Math 482 Workshop

Week 10: Integer Linear Programs

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers. Consider the integer linear program (ILP) below.

$$\text{maximize } z = 3x_1 + 2x_2 \tag{1a}$$

$$\text{subject to } 2x_1 + 2x_2 \leq 5, \tag{1b}$$

$$x_1 + x_2 \leq 4, \tag{1c}$$

$$x_1, x_2 \in \mathbb{Z}_{\geq 0} \tag{1d}$$

- I. Write the LP relaxation of this ILP.
- II. Solve the LP relaxation using the simplex method in matrix form. On each iteration, identify the basis, parameter set, basic solution, reduced cost, dual vector, primal slacks, and dual slacks.
- III.
 - a. Use the fractional optimal solution to generate a Gomory fractional cut.
 - b. Add the cut to the LP relaxation and show that the new LP only has one relevant (non-redundant) constraint.
 - c. Find an integer optimal solution to the new LP.
 - d. Explain how the optimal dual vectors to each relaxation LP can be used to provide an integer certificate of optimality.
- IV.
 - a. Use the fractional optimal solution to branch the LP into two subproblems.
 - b. One of these two subproblems is infeasible. Explain why. Then, use Gurobi to solve for the optimal solution of the feasible subproblem.
 - c. The current node has a fractional optimal solution. Branch this node into two subproblems.
 - d. Draw the feasible region for both of these subproblems. Use your drawing to identify the optimal solutions to each subproblem.
 - e. Identify the integer optimal solution to the original ILP.
 - f. Explain how the branch-and-bound tree provides an integer certificate of optimality.