

Math 482 Workshop

Week 6: Duality Certificates and Complementary Slackness

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers.

I. Consider the primal linear program P shown below.

$$\begin{aligned} \text{maximize} \quad & z = 2x_1 + 4x_2 + 5x_3 + 8x_4 \\ \text{subject to} \quad & x_1 + x_3 + 4x_4 \leq 5, \\ & -x_1 + 4x_2 + 2x_3 - 3x_4 \leq 22, \\ & 3x_1 + x_2 + x_3 + x_4 \leq 8, \\ & x_j \geq 0, \quad \forall j \in \{1, 2, 3, 4\} \end{aligned}$$

a. Write the dual linear program D .

b. Suppose that

$$\mathbf{x}^* = (0, 3, 5, 0)^T \quad \text{and} \quad \mathbf{y}^* = (1, 0, 4)^T.$$

Verify that \mathbf{x}^* is P -feasible and that \mathbf{y}^* is D -feasible.

c. Verify the complementary slackness conditions for $(\mathbf{x}^*, \mathbf{y}^*)$.

d. In part (c), you should find a complementary slackness condition where *both* sides of the “either–or” statement hold. Identify which condition(s) this happens for.

II. Consider the primal linear program P shown below.

$$\begin{aligned} \text{maximize} \quad & z = 13x_1 + 20x_2 + 17x_3 \\ \text{subject to} \quad & 8x_1 + 7x_2 + 9x_3 \leq 455, \\ & 5x_1 - x_2 + 6x_3 \leq 190, \\ & 4x_1 + 8x_2 - x_3 \leq 205, \\ & -x_1 + 2x_2 + 3x_3 \leq 80, \\ & -3x_1 - 5x_2 - 4x_3 \leq -200, \\ & x_j \geq 0, \quad \forall j \in \{1, 2, 3\} \end{aligned}$$

a. Write the dual linear program D .

b. Use weak duality and complementary slackness to decide whether either of the following pairs $(\mathbf{x}', \mathbf{y}')$ is a primal–dual optimal pair.

- (i) $\mathbf{x}' = \frac{1}{79}(0, 2300, 2205)^T$, $\mathbf{y}' = \frac{1}{79}(156, 0, 61, 0, 0)^T$.
- (ii) $\mathbf{x}' = \frac{1}{180}(4600, 2600, 2300)^T$, $\mathbf{y}' = \frac{1}{180}(0, 753, 546, 0, 0)^T$.
- (iii) $\mathbf{x}' = \frac{1}{275}(4895, 5280, 5445)^T$, $\mathbf{y}' = \frac{1}{275}(390, 0, 230, 465, 0)^T$.