# Math 482 Workshop

## Week 8: Matrix Form and Network Problems

**Instructions.** Write clear solutions on your own paper. Show enough work to justify your answers.

I. Let $G = (V, E)$ be a graph and consider the matching problem below.

$$\text{maximize} \quad z = \sum_{e \in E} x_e \tag{1a}$$

$$\text{subject to} \quad \sum_{e \in E:\, v \in e} x_e \leq 1, \ \forall v \in V, \tag{1b}$$

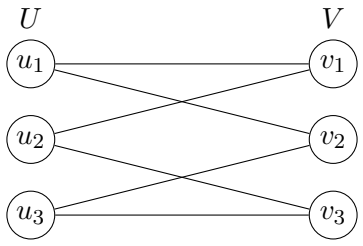
$$x_e \in \{0, 1\}, \ \forall e \in E \tag{1c}$$

a. If we relax the integrality constraint so that $0 \leq x_e \leq 1$ for all $e \in E$, then the resulting linear program is call the fractional matching LP. What does aa optimal solution to the fractional matching problem describe on the graph $G$?

b. Write the dual of the fractional matching problem.

c. If we restrict the dual variables to be integral, what problem is the dual modeling?

II. Consider the graph sown below




a. Write the matching model for this graph.

b. Let $A$ denote the coefficient matrix for this matching model. Note that $[A|I][]$ is totally unimodular, that is, every square submatrix has determinant $-1$, 0, or 1. Why does this imply that the solution to the fractional matching problem will be integral?

c. Find an optimal solution to the fractional matching problem using the simplex method in matrix form. On each iteration, identify the basis, parameter set, basic solution, reduced cost, dual vector, primal slacks, and dual slacks.

d. Verify optimality by showin that the complementary slackness conditions are satisfied.

III. Consider the linear ordering problem described in (4a)–(4d) of the Network Model notes. The decision variables $x_{ij} \in \{0, 1\}$ form a subdiagraph.

(a) A tournament is a directed graph with exactly one edge between every pair of vertices. Explain why constraint (4b) ensures that the subdigraph is a tournament.

(b) An acyclic digraph is one with no cycles. Explain why constraints (4b)–(4c) ensures that the subdigraph is acyclic.