Graph Theory

Homework 5

Due: March 18, 2023

1 Problems

I. Let $G = (V, E)$ and $G' = (V', E')$ be isomorphic graphs. Prove that there is a permutation matrix P such that for each $A \in \mathcal{S}(G)$, $PAP^{-1} \in \mathcal{S}(G')$.

II. Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix. Prove that A has n real eigenvalues, counting multiplicities.

III. Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix. Prove that eigenvectors corresponding to distinct eigenvalues are orthogonal.

IV. Prove that the Laplacian matrix associated with any graph $G = (V, E)$ is an M-matrix.

V. Let L denote the Laplacian matrix associated with a graph $G = (V, E)$ of order $n \geq 1$. Prove that the eigenvalues of L are contained in the interval $[0, n]$. Explain why no proper sub-interval of $[0, n]$ will work in general.