

Graph Theory

Homework 6

Due: March 22, 2023

1 Problems

I. Let $G = (V, E)$ be a simple graph of order $n \geq 1$ and let L denote the Laplacian matrix of G . Prove that $\sigma(L) = \{0, 1, \dots, 1, n\}$ if and only if G is isomorphic to the star graph of order n .

II. Let T be a tree of order $n \geq 3$. Prove that the algebraic connectivity satisfies

$$0 < a(T) \leq 1.$$

III. Let $G = (V, E)$ be a simple graph of order $n \geq 1$. Prove that the algebraic connectivity satisfies

$$a(G) \leq n - \alpha(G),$$

where $\alpha(G)$ denotes the independence number of G .