

Graph Theory

Thomas R. Cameron

January 8, 2024

1 Key Topics

Today we introduce several classical problems that are best viewed as a graph. For more information, see [1, Chapter 1].

1.1 Seven Bridges of Königsberg

In the late 1700s, in the city of Königsberg, there were seven bridges connecting various parts of the city as shown in Figure 1.

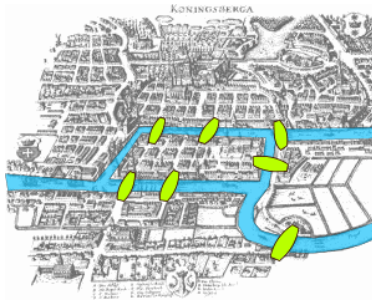


Figure 1: Seven Bridges of Königsberg

The townspeople of Königsberg enjoyed strolling through the city and many wondered if it was possible to cross every bridge exactly once during a single walk. In mathematics, we like to use diagrams to help illustrate problems. For example, one may use the graph in Figure 2 to illustrate the seven bridges of Königsberg problem, where the green dots (vertices) are the parts of the city and the black lines (edges) are the bridges. In particular, the seven bridges of Königsberg problem may be phrased as follows: Is there a walk on the graph in Figure 2 that uses every edge exactly once?

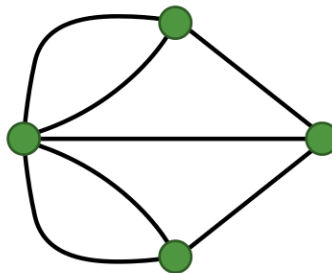


Figure 2: Graph of Königsberg

1.2 Map Coloring

Consider the map of a continent with several countries shown in Figure 3

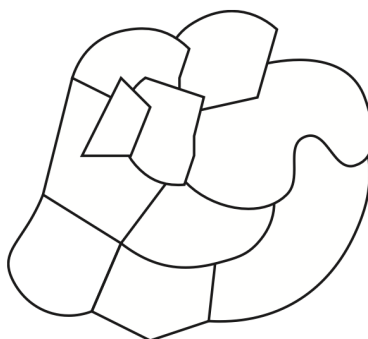


Figure 3: A Mythical Continent

As a cartographer, you are charged with designing a map of the continent in Figure 3. To distinguish each country you want to use different colors; however, if you make every country a different color the map would be garish. Therefore, you decide to use the smallest number of colors possible so that neighboring countries do not share the same color.

2 Exercises

References

- [1] D. JOYNER, M. V. NGUYEN, AND D. PHILLIPS, *Algorithmic Graph Theory and Sage*, 2013.